ERRATA for T. Shifrin's *Multivariable Mathematics: Linear Algebra, Multivariable Calculus, and Manifolds*

All of these except those marked with ($\star$) have been corrected in the second printing (June, 2017).

($\star$) **p. 41**, Exercise **17b**. The exponent should be k, an arbitrary positive integer.

p. 47, line **11**. In the rightmost determinant, the first entry of the second column should be z_1.

($\star$) **p. 66**, Example 1**c**. The rectangle, as given, may in fact not be contained in S, as the lower edge or left edge may go out of the first quadrant. Let's choose $\tilde{\delta} = \min(\delta, a, b)$ to be safe.

p. 79, Exercise **4**. x should be $\mathbf{x}$ throughout.

p. 86, Exercise **12b**. Here $\mathbf{f}: \mathcal{M}_{n\times n} \to \mathbb{R}$.

p. 91, Example **5**. On the second line it should be $\mathbf{f}: \mathbb{R}^2 - \{y = 0\} \to \mathbb{R}^2$.

p. 92, Example **7**. The reference should be to Example **3** of Chapter 2, Section 3.

p. 93, Proposition **2.4**, ff. Standard terminology is that a function f is $\mathcal{C}^1$ if f and its partial derivatives are continuous. Note that in the proof of the Proposition, since the partial derivatives exist, we get continuity of f along horizontal and vertical lines, which is all we need to apply the Mean Value Theorem. Thus, the Proposition is correct as stated.

p. 103, Exercise **6**. The symbol for liter (l) looks too much like a 1. For clarity, it would help to change these to ℓ.

p. 103, Exercise **11**. Prove that a *differentiable* function f is homogeneous ...

($\star$) **p. 121**, line **–5**. $r(b + k) - r(b)$.

p. 145, Exercise **13**. In (**b**) and (**d**) the vectors $\mathbf{b}$ and $\mathbf{b}_i$ should be nonzero.

p. 155, Exercise **1**. ...find a product of elementary matrices $E = \cdots E_2E_1$ so that EA is in echelon form.

p. 185, Exercise **6a**. nonzero matrix A.

($\star$) **p. 199**, Theorem **1.2**. X should be nonempty.

($\star$) **p. 201**, Exercise **8**. S should be nonempty.

pp. 202, Lemma **2.1**. $Df(\mathbf{a}) = \mathrm{O}$ ...

($\star$) **p. 203**, lines **8, 13**. $Df(\mathbf{a}) = \mathrm{O}$.

p. 203, **Definition**. A critical point $\mathbf{a}$ is a saddle point if for every $\delta > 0$, there are points $\mathbf{x}, \mathbf{y} \in B(\mathbf{a}, \delta)$ with $f(\mathbf{x}) < f(\mathbf{a})$ and $f(\mathbf{y}) > f(\mathbf{a})$.

p. 207, Exercise **2**. The opposite corner should also be in the first octant, i.e., should have x, y, and z all positive.

p. 225, Exercise **33**. ... marginal productivity *per dollar* ...

p. 225, Exercise **34**. On line **2**, $Dg(\mathbf{a}) \neq \mathrm{O}$.

($\star$) **p. 233**, Lemma **5.1**. Suppose $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ is a basis for V. Then the equation

$$\mathbf{x} = \sum \mathrm{proj}_{\mathbf{v}_i} \mathbf{x} = \dots$$

holds for all $\mathbf{x} \in V$ if and only if ...

p. 250, footnote. Kantorovich.

($\star$) **p. 252**, proof of Theorem **2.1**. Add to the end of line 2 of the proof: "and since the norm of a linear map is a continuous function (See Exercise 5, p. 201)" ...

p. 256, line **6**. Z is a neighborhood of $\begin{bmatrix} \mathbf{x}_0 \\ \mathbf{0} \end{bmatrix}$. In Figure 2.4, Z should be slid to the right, containing $V \times \{\mathbf{0}\}$.

($\star$) **p. 258**, line **3**. Delete the first "as."

p. 261, Exercise **13a**. Suppose $f\begin{pmatrix} \mathbf{x}_0 \\ t_0 \end{pmatrix} = \dfrac{\partial f}{\partial t}\begin{pmatrix} \mathbf{x}_0 \\ t_0 \end{pmatrix} = 0$ and the matrix ... is nonsingular. Show that for some $\delta > 0$, there is a $\mathcal{C}^1$ curve $\mathbf{g}\colon (t_0 - \delta, t_0 + \delta) \to \mathbb{R}^2$ with $\mathbf{g}(t_0) = \mathbf{x}_0$ so that ...

p. 271, Proposition **1.6**. R' and R'' should overlap in only a "face," not in a proper subrectangle.

p. 272, line **5**. If X contains a frontier point of R, we cannot ensure this; however, the closure of Y will still be disjoint from X and the argument continues fine.

p. 275, Exercise **10**. $R \subset \mathbb{R}^n$; line **5** ... requires at most volume $2A\delta$.

p. 276, Exercise **15b**. $D = \{\mathbf{x} \in R : f \text{ is discontinuous at } \mathbf{x}\}$.

p. 316, line **–3**. Proof of Proposition 5.**14**.

p. 322, Exercise **10d**. The problem should ask only for an example when A and C do not commute. In fact, using the continuity of det, the astute reader should be able to check that the result of part c *does* hold whenever A and C commute.

($\star$) **p. 326**, Proof of **Theorem 6.4**. In the proof of Theorem 6.4, the reduction to a rectangle is not valid. We have to cover Ω with a union R of rectangles (with rational sidelengths) contained in U. This can then be partitioned into cubes and the proof proceeds.

p. 328, lines **13–15**. In the long inequality we should have $\varepsilon \,\mathrm{vol}\,(R)\big(1 + Mn\big)$ and $\varepsilon \,\mathrm{vol}\,(R)\big(2^n + 2^{n-1}Mn\big)$. Then let $\beta = \mathrm{vol}\,(R)\big(2^n + 2^{n-1}Mn\big)$.

p. 329, line **1**. Section 3, not section 4.

($\star$) **p. 345**, lines **4–5**. We need the remark here that $\mathbf{g}_2^{-1} \circ \mathbf{g}_1$ is smooth. This can be proved by what should be an exercise in §6.3: Using the notation of part **3** of the Definition on p. 262 of a k-dimensional manifold, perhaps shrinking W, there is a smooth function $\mathbf{h}\colon W \to U$ whose restriction to $M \cap W$ is $\mathbf{g}^{-1}$. (Hint:

Without loss of generality, assume $\mathbf{g}(\mathbf{u}_0) = \mathbf{p}$ and write $\mathbf{g}(\mathbf{u}) = \begin{bmatrix} \mathbf{g}_1(\mathbf{u}) \\ \mathbf{g}_2(\mathbf{u}) \end{bmatrix} \in \mathbb{R}^k \times \mathbb{R}^{n-k}$, where $D\mathbf{g}_1(\mathbf{u}_0)$ is nonsingular.)

p. 352, add to **Remark**: Also, note that we are using the notation $\oint_C \omega$ to denote the integral of ω around the closed curve (or loop) C. This notation is prevalent in physics texts.

p. 355, lines **−2** and **−1**. a should be $\mathbf{a}$.

p. 368–369, Example **2**. In parts a and c, $D = (0,1) \times (0, 2\pi)$.

p. 380, line **8**. Add: "parametrization $\mathbf{g}\colon U \to \mathbb{R}^n$ with $U \subset \mathbb{R}^k_+$ and"

p. 381, **last** line. $\mathbf{g}_i\colon B(\mathbf{0}, 2) \to V_i$, and $V_i' = \mathbf{g}_i(B(\mathbf{0}, 1)) \subset V_i$ cover M.

p. 382, line **12**. Delete the last equality in the displayed string of equations.

p. 410, lines **4** and **5**. All the integrals should be over S^{2m}.

p. 411, Exercise **9**. Suppose $U \subset \mathbb{C}$ is open, $f, g\colon U \to \mathbb{C}$ are smooth, and $C \subset U$ is a closed curve. Suppose that on C we have $f, g \neq 0$ and $|g - f| < |f|$. Prove that ...

($\star$) **p. 426**, footnote. The fastidious reader should instead observe that this follows directly from Proposition 5.18 of Chapter 7.

p. 433, line **5**. The 22 entry of $B - I$ should be 2.

p. 444, Example **7**, line **–3**. $\dot{x}_1 = -x_2$.

p. 445, Example **8**. Delete the first "the" in the first line.

($\star$) **p. 454**, Exercise **17c**. The result of Exercise 9.2.22 is needed to provide the suggested continuity argument, as well. We should insert a remark that the result of c holds even when the eigenvalues are complex. This is needed for #19.

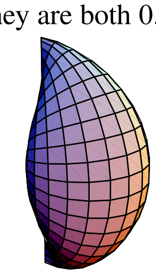
p. 457, lines **11–12**. "Let $W = (\text{Span}\,(\mathbf{v}_1))^\perp \subset \mathbb{R}^n$" should precede the second sentence of the paragraph.

p. 476, **#2.2.13**. min should be max.

p. 480, **#4.5.11a**. $DF(\mathbf{x})$ has rank 2 at every point $\mathbf{x} \in M$: Either $x_1 = x_2$ and $x_3 = -x_4$ or $x_1 = -x_2$ and $x_3 = x_4$, so x_1x_2 and x_3x_4 have opposite signs unless they are both 0.

p. 482, **#6.2.1**: $D\mathbf{g}(\mathbf{f}(\mathbf{x}_0)) = \dfrac{1}{2(x_0^2 + y_0^2)} \dots$

p. 483, **#7.3.12**: The picture is not correct.



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