By providing my signature below, I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): Solutions Name (print): ____________

Student Number: ____________

Instructor's Name: ____________ Class Time: ____________

Problem Number	Points Possible	Points Earned
1	20	
2	10	
3	15	
4	28	
5	15	
6	20	
7	14	
8	10	
9	14	
10	20	
11	16	
12	12	
13	15	
14	25	
Total:	234	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion, then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- You may not use a cell phone or smart watch. Please turn off your cell phones and store them and your smart watches away from your desk or table.
- You are only allowed to use a **TI-30XS Multiview** calculator. No other calculators are permitted, and sharing of calculators is not permitted.
- You do not have to use a calculator; answers containing symbolic expressions such as $\cos(\pi/3)$ and $\ln(e^4)$ are acceptable. Include an exact answer for each problem.

1. Determine the following limits; briefly explain your thinking on parts (b) and (c). If you apply L'Hopital's rule, indicate where you have applied it and why you can apply it. Print your final answer in the box provided.

(a) [6 pts] no explanation needed: $\lim_{x\to -1} \left(x^7 - 4x + 2\right)$

$= (-1)^7 - 4(-1) + 2$ ← fine to stop here
$= -1 + 4 + 2$
$= 5$

answer: 5

(b) [6 pts] $\lim_{x\to -\infty} \frac{x-1}{|x-1|}$ $= -1$

possible explanations:

① $\frac{x-1}{|x-1|} = -1$ for $x<1$

② $x-1 = -|x-1|$ for $x<1$

answer: -1

(c) [8 pts] $\lim_{x\to 0} \frac{e^{5x}-1}{3x}$ $\overset{LH}{=} \lim_{x\to 0} \frac{5e^{5x}}{3} = \frac{5e^0}{3} = \frac{5}{3}$

If $\frac{0}{0}$

answer: $\frac{5}{3}$

2. [10 pts] Determine whether the function below is continuous at $x = 3$.
Use the definition of continuity to explain your answer.

$$f(x) = \begin{cases} 4x - 6 & \text{if } x \le 3 \\ \dfrac{\sqrt{x} - \sqrt{3}}{x - 3} & \text{if } x > 3 \end{cases}$$

Method 1

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (4x - 6) = 12 - 6 = 6$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{\sqrt{x} - \sqrt{3}}{x - 3} \cdot \frac{\sqrt{x} + \sqrt{3}}{\sqrt{x} + \sqrt{3}} = \lim_{x \to 3^+} \frac{x - 3}{(x - 3)(\sqrt{x} + \sqrt{3})} = \lim_{x \to 3^+} \frac{1}{\sqrt{x} + \sqrt{3}} = \frac{1}{2\sqrt{3}}$

Since $\lim_{x \to 3} f(x)$ does not exist, we know f is not continuous at $x = 3$.

Method 2

$f(3) = 6$

$\lim_{x \to 3^+} \frac{\sqrt{x} - \sqrt{3}}{x - 3} \cdot \frac{\sqrt{x} + \sqrt{3}}{\sqrt{x} + \sqrt{3}} = \lim_{x \to 3} \frac{x - 3}{(x - 3)(\sqrt{x} + \sqrt{3})} = \lim_{x \to 3} \frac{1}{\sqrt{x} + \sqrt{3}} = \frac{1}{2\sqrt{3}}$

Since $f(3) \neq \lim_{x \to 3^+} f(x)$, we know f is not continuous at $x = 3$.

3. (a) [5 pts] State the limit definition of the derivative of $f(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \qquad \underline{\text{OR}} \qquad f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

(b) [10 pts] Use the **limit definition of the derivative** to determine the derivative of $f(x) = \dfrac{1}{1+2x}$. (No points will be awarded for the application of differentiation rules.)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \left(\frac{1}{h}\left(\frac{1}{1+2(x+h)} - \frac{1}{1+2x}\right)\right)$$

$$= \lim_{h\to 0} \left[\frac{1}{h}\left(\frac{(1+2x)-(1+2x+2h)}{(1+2x+2h)(1+2x)}\right)\right]$$

$$= \lim_{h\to 0} \left[\frac{1}{h}\cdot\frac{-2h}{(1+2x+2h)(1+2x)}\right]$$

$$= \lim_{h\to 0} \frac{-2}{(1+2x+2h)(1+2x)}$$

$$= \frac{-2}{(1+2x)^2}$$

OR

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{1}{x-a}\left(\frac{1}{1+2x} - \frac{1}{1+2a}\right)$$

$$= \lim_{x\to a} \frac{1}{x-a}\cdot\left(\frac{1+2a-(1+2x)}{(1+2x)(1+2a)}\right)$$

$$= \lim_{x\to a} \frac{1}{x-a}\left(\frac{2a-2x}{(1+2x)(1+2a)}\right)$$

$$= \lim_{x\to a} \frac{1}{x-a}\cdot\frac{-2(x-a)}{(1+2x)(1+2a)}$$

$$= \lim_{x\to a} \frac{-2}{(1+2x)(1+2a)}$$

$$= \frac{-2}{(1+2a)^2}$$

4. Determine the first derivative of each of the following functions. Print your answer in the box provided. **You do not have to simplify your answers or explain your steps.**

______ (a) [6 pts] $f(x) = 7x^5 - x^{3/5} + 2018$

$f'(x) = 35x^4 - \frac{3}{5}x^{-2/5}$

______ (b) [6 pts] $g(x) = \dfrac{e^x}{x^2+5} = e^x(x^2+5)^{-1}$

$g'(x) = \dfrac{(x^2+5)e^x - e^x(2x)}{(x^2+5)^2} = \dfrac{e^x(x^2-2x+5)}{(x^2+5)^2}$

or $g'(x) = e^x \cdot -1(x^2+5)^{-2} \cdot 2x + (x^2+5)^{-1} \cdot e^x$

______ (c) [8 pts] $f(x) = x^9 \cos(4x)$

$f'(x) =$
$x^9 \cdot -\sin(4x) \cdot 4 + \cos(4x) \cdot 9x^8$

______ (d) [8 pts] $f(x) = \arctan(x) + \sqrt{x^7 + 3x}$

$f'(x) = \dfrac{1}{1+x^2} + \frac{1}{2}(x^7+3x)^{-1/2} \cdot (7x^6+3)$

5. (a) [10 pts] Determine $\dfrac{dy}{dx}$ where $2x^3 + 3x^2y + y^3 = 4$.
Print your answer in the box provided. You do not have to simplify your answer.

$$6x^2 + 3x^2\frac{dy}{dx} + 6xy + 3y^2\frac{dy}{dx} = 0$$
$$3x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -6x^2 - 6xy$$
$$\frac{dy}{dx}\left(3x^2 + 3y^2\right) = -6x^2 - 6xy$$
$$\frac{dy}{dx} = \frac{-6x^2 - 6xy}{3x^2 + 3y^2} = \frac{-2x(x+y)}{x^2+y^2}$$

$$\frac{dy}{dx} = -\frac{2x(x+y)}{x^2+y^2}$$

(b) [5 pts] Determine all points on the graph of $2x^3 + 3x^2y + y^3 = 4$ at which the curve has a horizontal tangent line. Give your answer(s) in the form (x, y). (You may use your work from part (a).)

$$-2x(x+y) = 0$$

$$-2x = 0$$
$$x = 0$$
$$y^3 = 4$$
$$y = \sqrt[3]{4} = 2^{2/3}$$

$$x + y = 0$$
$$y = -x$$
$$2x^3 - 3x^3 - x^3 = 4$$
$$-2x^3 = 4$$
$$x^3 = -2$$
$$x = \sqrt[3]{-2} = -\sqrt[3]{2}$$
$$y = -\sqrt[3]{-2} = \sqrt[3]{2}$$

Answer: $(0, 2^{2/3})$ $(-\sqrt[3]{2}, \sqrt[3]{2})$

6. Determine the following indefinite integrals. Print your answer to each part in the box provided. **You do not have to simplify your answers or explain your steps.**

______ (a) [6 pts] $\int \left(x^3 - 7x + 12\right)\, dx$

Final answer:
$\frac{1}{4}x^4 - \frac{7}{2}x^2 + 12x + C$

______ (b) [6 pts] $\int \left(\frac{1}{\sqrt{1-x^2}} + \sec(x)\tan(x)\right)\, dx$

Final answer:
$\arcsin(x) + \sec(x) + C$ or $-\arccos(x) + \sec(x) + C$

______ (c) [8 pts] $\int e^{\sin(3x)}(\cos(3x))\, dx = \int e^u \cdot \frac{1}{3} du = \frac{1}{3}e^u + C = \frac{1}{3}e^{\sin(3x)} + C$

$u = \sin(3x)$
$du = 3\cos(3x)dx$
$\frac{1}{3}du = \cos(3x)dx$

Final answer: $\frac{1}{3}e^{\sin(3x)} + C$

7. Evaluate the following definite integrals. Print your answer in the box provided.
You do not have to simplify your answers or explain your steps.

(a) [6 pts] $\int_0^{1/2} (e^x - \cos(\pi x))\ dx$

$$= \left[e^x - \frac{1}{\pi}\sin(\pi x)\right]_0^{1/2}$$

$$= \left(e^{1/2} - \frac{1}{\pi}\sin(\pi/2)\right) - \left(e^0 - \frac{1}{\pi}\cancel{\sin(0)}\right)$$

$$= e^{1/2} - \frac{1}{\pi} - 1$$

Value: $e^{1/2} - \frac{1}{\pi} - 1$

(b) [8 pts] $\int_0^1 \left(\frac{1}{x^6 - 4x + 7}\right)(6x^5 - 4)\ dx$ $= \int_7^4 \frac{1}{u}\,du = \left[\ln|u|\right]_7^4 = \ln(4) - \ln(7)$

$u = x^6 - 4x + 7$
$du = (6x^5 - 4)dx$
$u(0) = 7$
$u(1) = 1 - 4 + 7 = 4$

Value: $\ln(4) - \ln(7) = \ln(4/7)$

8. [10 pts] Use calculus to determine the absolute maximum and absolute minimum values of the function below on the interval $[1, 5]$:

$$f(x) = x^3(x-4)^3$$

$$f'(x) = x^3 \cdot 3(x-4)^2 + (x-4)^3 \cdot 3x^2$$
$$= 3x^2(x-4)^2 \cdot (x + (x-4))$$
$$= 3x^2(x-4)^2(2x-4)$$
$$= 3x^2 \cdot (x-4)^2 \cdot 2(x-2)$$

f' dne: none $f' = 0$ at $x = \cancel{0}, 2, 4$

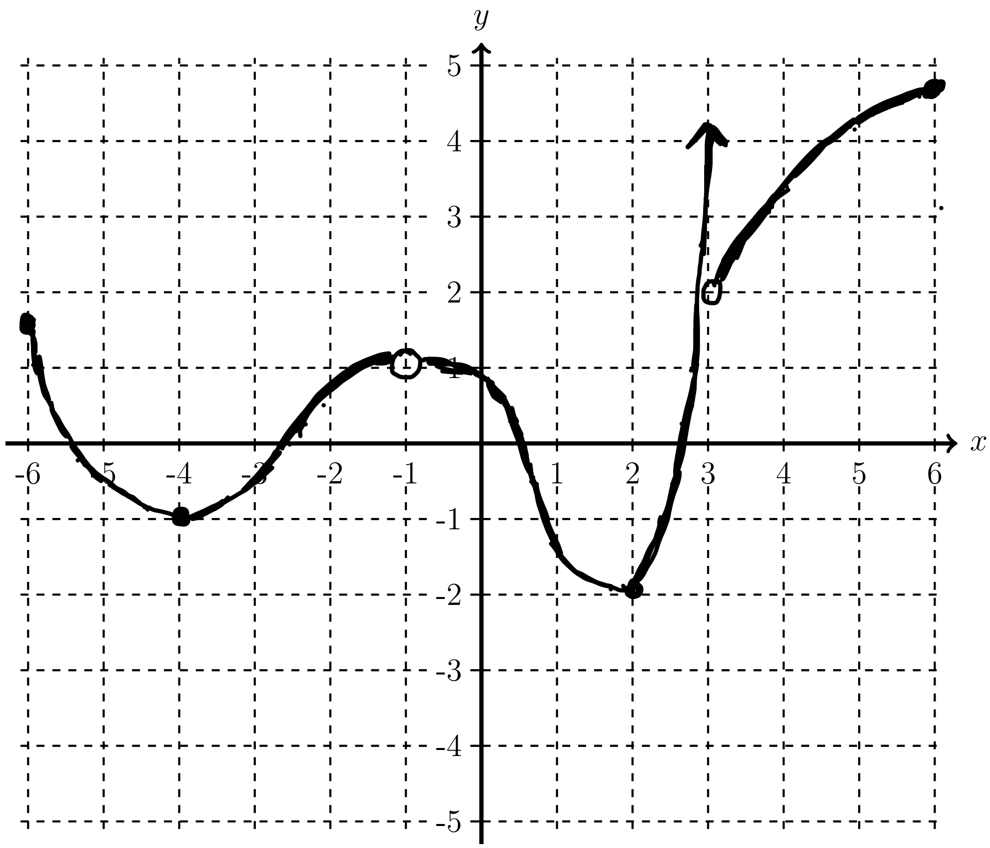
domain

x	$f(x)$
1	$1 \cdot -27 = -27$
2	$8 \cdot -8 = -64 \leftarrow$ abs min
4	0
5	$125 \leftarrow$ abs max

9. [14 pts] Sketch the graph of a function f satisfying all of the following properties:

- The domain of f is $[-6,-1)\cup(-1,3)\cup(3,6]$
- $\lim_{x\to -1} f(x) = 1$
- $\lim_{x\to 3^-} f(x) = \infty$ and $\lim_{x\to 3^+} f(x) = 2$
- $f(-4) = -1$ and $f(2) = -2$
- f is continuous at every point in its domain
- The increasing/decreasing behavior of f is given in the following table:

interval	$[-6,-4)$	$(-4,-1)$	$(-1,2)$	$(2,3)$	$(3,6]$
f	decreasing	increasing	decreasing	increasing	increasing



10. Consider the function $f(x) = e^{x^2}$.

(a) [10 pts] Determine interval(s) on which f is increasing and interval(s) on which f is decreasing.

$$f'(x) = e^{x^2} \cdot 2x$$

$$0 = e^{x^2} \cdot 2x$$

$$x = 0$$

	$(-\infty, 0)$	$(0, \infty)$
f'	$f'(-1) = -2e$	$f'(1) = 2e$
sign f'	$-$	$+$
beh. f	decreasing	increasing

(b) [6 pts] Approximate the integral $\int_1^3 f(x)\,dx$ by a Riemann sum using four sub-intervals of equal width and left endpoints. You do not need to simplify your expression for the Riemann sum; expressions involving powers of e such as $e^{\sqrt{2}}$ are acceptable.

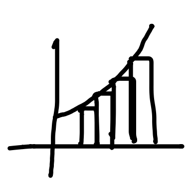
$n = 4$

$\Delta x = \frac{3-1}{4} = \frac{1}{2}$

$[1, 1.5]\,[1.5, 2]\,[2, 2.5]\,[2.5, 3]$

$$\boxed{e^1 \cdot \tfrac{1}{2} + e^{(1.5)^2} \cdot \tfrac{1}{2} + e^{(2^2)} \cdot \tfrac{1}{2} + e^{(2.5)^2} \cdot \tfrac{1}{2}}$$

(c) [4 pts] Based only on your answer to part (a), will your final answer to (b) be an over-estimate or under-estimate of the true exact value of $\int_1^3 f(x)\,dx$? Explain briefly.



f is increasing on $[1,3]$ so left endpoints yield an underestimate

11. Consider the function $g(x) = x^2 + 4$.

(a) [8 pts] Determine the linearization $L(x)$ of $g(x)$ at $x = 2$.

$L(x) = g(a) + g'(a)(x-a)$; $g'(x) = 2x$

$a = 2$

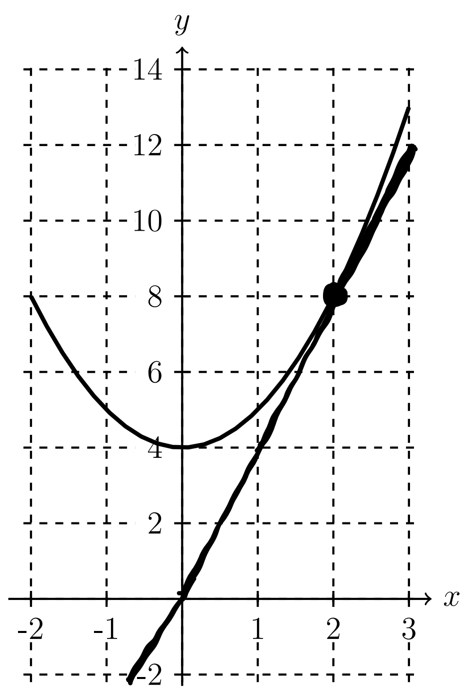
$g(2) = 8$

$g'(2) = 2 \cdot 2 = 4$

$L(x) = 8 + 4(x-2)$

or $L(x) = 4x$

(b) [4 pts] Sketch the graph of $y = L(x)$ on the graph of $y = g(x)$ below.

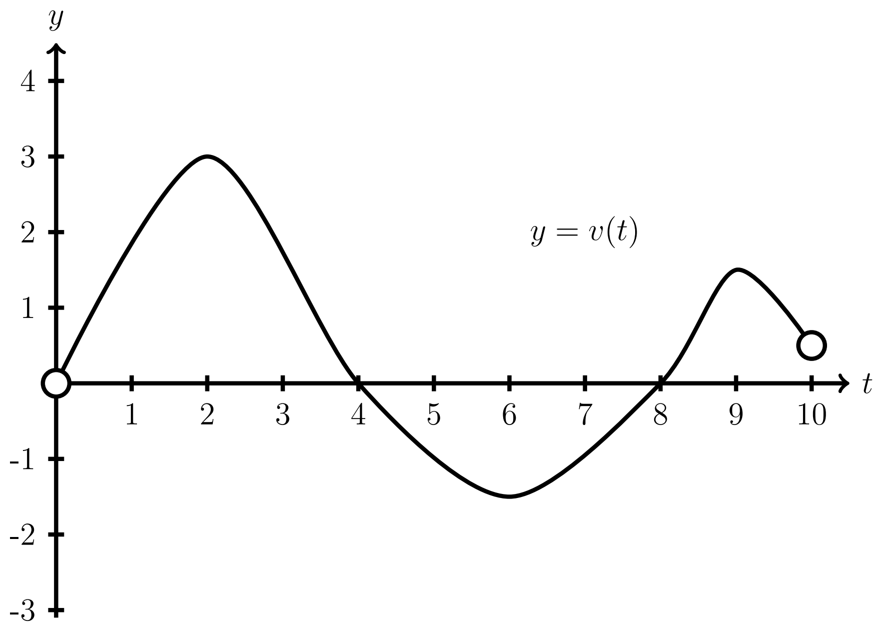


(c) [4 pts] Use $L(x)$ to estimate the value of $g(2.5)$.

$g(2.5) \approx L(2.5)$

$L(2.5) = 4(2.5) = 10$

12. An object moves along a straight line with position function $s(t)$, where s is in meters and t is in seconds with $0 < t < 10$. The graph below represents $v(t)$, the object's velocity function.



(a) [4 pts] Determine the intervals on which the object's position function $s(t)$ is increasing.

$(0,4], [8,10)$ or $(0,4), (8,10)$

(b) [4 pts] Determine intervals on which the graph of the object's position function $s(t)$ is concave upward.

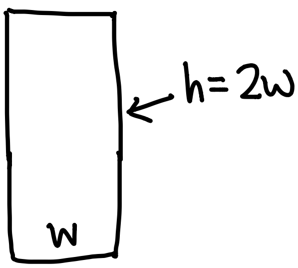
$(0,2], [6,9]$ or $(0,2), (6,9)$

(c) [4 pts] Determine all values of t at which the position function $s(t)$ has a relative minimum.

$t = 8$

13. [15 pts] The dimensions of a rectangle are changing, but the rectangle's height is always twice its width. When the area of the rectangle is 50 in^2, the area of the rectangle is increasing at a rate of 10 in^2 per second. Determine the rate of change of the perimeter of the rectangle at that instant.

goal: $\frac{dP}{dt}$



given information:

$h = 2w$; when $A = 50 \text{ in}^2$, $\frac{dA}{dt} = 10 \text{ in}^2/\text{sec}$

$P = 2w + 2h$

$P = 2w + 2(2w)$

$P = 6w$

$\frac{dP}{dt} = 6\frac{dw}{dt}$

$A = wh$

$A = w(2w)$

$A = 2w^2$

$\frac{dA}{dt} = 4w\frac{dw}{dt}$

$A = 50 = 2w^2$

$w = 5$ in

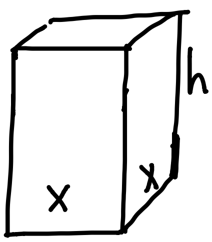
$10 = 4 \cdot 5 \cdot \frac{dw}{dt}$

$1/2 = dw/dt$

$$\boxed{\frac{dP}{dt} = 3 \text{ in}/\text{sec}}$$

14. You plan to make a box with a square base and lid, with largest possible volume, and with surface area 140 in^2. Note: The last part of this problem is on the next page.

____ (a) [10 pts] If the square base of the box is x inches wide, determine a formula for the function $V(x)$ representing the volume of the box (in cubic inches). Your function should contain the variable x and cannot contain any other variables.



$$V = x^2 h$$

$$140 = 2x^2 + 4xh$$

$$\frac{140-2x^2}{4x} = h$$

$$V = x^2\left(\frac{140-2x^2}{4x}\right)$$

$$V = \frac{1}{4}(140x - 2x^3)$$

$$V(x) = \frac{1}{4}(140x - 2x^3) = \frac{70}{2}x - \frac{1}{2}x^3$$

____ (b) [5 pts] Determine a domain for $V(x)$ which makes sense in the context of the scenario. Briefly explain why you chose the domain you provided.

$(0, \sqrt{70})$

To make sense, we need $x > 0$ and $h > 0$.

$$0 = \frac{140-2x^2}{4x}$$

$$0 = 140 - 2x^2$$

$$140 = 2x^2$$

$$70 = x^2$$

$$x = \sqrt{70}$$

Note: $(0, \sqrt{70}]$ is also appropriate if you allow a box with height 0

The problem continues on the next page.

Problem 14, continued:

(c) [10 pts] Determine the largest possible volume of such a box. You must use calculus to verify that the volume you find is the largest possible volume.

$V(x)=\frac{1}{4}(140x-2x^3)$

$V'(x)=\frac{1}{4}(140-6x^2)$

$0=140-6x^2$

$6x^2=140$

$x^2=140/6=70/3$

$x=\sqrt{70/3}$

① using $(0,\sqrt{70})$ for the domain:

2nd derivative test:

$V''(x)=\frac{1}{4}(-12x)$

$V''(\sqrt{70/3})$ is negative so $V(\sqrt{70/3})$ is the rel. max volume.

OR

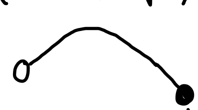
1st derivative test:

$V'(x)=140-6x^2$

	$(0,\sqrt{70/3})$	$(\sqrt{70/3},\sqrt{70})$
V'	$+$	$-$
V	increasing	decreasing

There's a rel. max at $x=\sqrt{70/3}$.

Since there's only one critical number in $(0,\sqrt{70})$, the rel. max is absolute. $V=\frac{1}{4}\left(140\sqrt{70/3}-2\left(\sqrt{70/3}\right)^3\right)$ in^3

② using $(0,\sqrt{70}]$ for the domain: use 1st or 2nd derivative test above to show that there's a rel max at $x=\sqrt{70/3}$. Since V is continuous on $(0,\sqrt{70}]$, the graph must look like 

That means there is an absolute max at $x=\sqrt{70/3}$.

The max volume is $V=\frac{1}{4}\left(140\sqrt{70/3}-2\left(\sqrt{\frac{70}{3}}\right)^3\right)$ in^3.

This page is extra space for work. **Do not detach this page.** If you want us to consider the work on this page, you should print your name, instructor and class meeting time below. For the problems where you want us to look at this work, please write "see last page" next to your work on the problem page so that we know to look here.

Name (print): ______________________ Instructor (print): ______________________

Class Meeting Time: ______________