By providing my signature below, I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (print): Solutions Name (sign): ____________

Student ID (81#): ____________

Instructor's Name: ____________ Class Time: ____________

Problem Number	Points Possible	Points Earned
1	16	
2	20	
3	10	
4	26	
5	18	
6	18	
7	10	
8	14	
9	14	
10	18	
Total:	164	

- If you need extra space, use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion, then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Cell phones and smart watches are NOT allowed; smart devices (including smart watches and cell phones) may not be on your person and must be stored in a backpack, purse, or other storage item left at the front of the classroom.
- You are only allowed to use a **TI-30XS Multiview** calculator; the name must match exactly. No other calculators are permitted, and sharing of calculators is not permitted.
- You do not have to use a calculator; answers containing symbolic expressions such as $\cos(\pi/3)$ and $\ln(e^4)$ are acceptable. Include an exact answer for each problem.

1. Determine the following limits. If you apply L'Hopital's rule, indicate where you have applied it and why you can apply it. If your final answer is "does not exist," ∞, or $-\infty$, briefly explain your answer. (You will not receive full credit for a "does not exist" answer if the answer is ∞ or $-\infty$.)

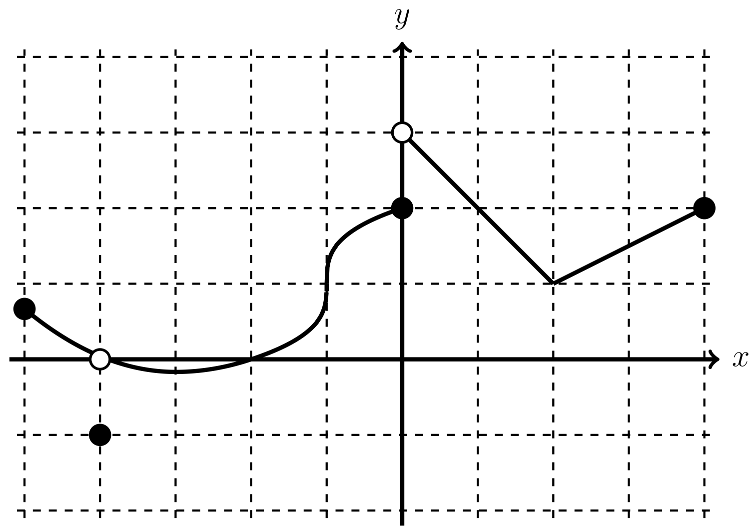
   (a) [4 pts] $\lim_{x\to 5} \left(4 + 7x - x^2\right) = 4+35-25 = \boxed{14}$

   (b) [6 pts] $\lim_{x\to 0} \frac{e^{3x}-1}{7x} \overset{\text{LH}}{=} \lim_{x\to 0} \frac{3e^{3x}}{7} = \boxed{\frac{3}{7}}$

   IF $\frac{0}{0}$

   (c) [6 pts] $\lim_{x\to\infty} \frac{7x^5-3x^2+1}{2x^5+x^2-6} = \lim_{x\to\infty} \frac{x^5\left(7-\frac{3}{x^3}+\frac{1}{x^5}\right)}{x^5\left(2+\frac{1}{x^3}-\frac{6}{x^5}\right)} = \lim_{x\to\infty} \frac{7-3/x^3+1/x^5}{2+1/x^3-6/x^5} = \boxed{7/2}$

2. Consider the graph of $y = f(x)$ given below. The grid lines are one unit apart. Based on the graph answer the following questions.



(a) [4 pts] Determine $\lim_{x\to 2} f(x)$. 1

(b) [4 pts] Determine $f(0)$. 2

(c) [6 pts] Determine all values of x in the interval $(-5, 4)$ at which $f(x)$ is NOT continuous. Briefly explain your thinking.

x = -4, 0

x = -4: $f(-4) \neq \lim_{x\to -4} f(x)$

x = 0: $\lim_{x\to 0} f(x)$ does not exist

(d) [6 pts] Determine all values of x in the interval $(-5, 4)$ at which $f'(x)$ is undefined. Briefly explain your thinking.

x = -4, -1, 0, 2

x = -4 and x = 0: f is not continuous there

x = -1: vertical tangent, x = 2: graph has a corner

3. (a) [2 pts] State the limit definition of the derivative of $f(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \quad \text{or} \quad f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

(b) [8 pts] Use the **limit definition of the derivative** to determine the derivative of $f(x) = \sqrt{8-3x}$. No points will be awarded for the application of differentiation rules (and L'Hopital's rule is not allowed).

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{8-3(x+h)} - \sqrt{8-3x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{8-3x-3h} - \sqrt{8-3x}}{h} \cdot \frac{\sqrt{8-3x-3h} + \sqrt{8-3x}}{\sqrt{8-3x-3h} + \sqrt{8-3x}}$$

$$= \lim_{h\to 0} \frac{(\cancel{8}\cancel{-3x}-3h)-(\cancel{8-3x})}{h(\sqrt{8-3x-3h} + \sqrt{8-3x})}$$

$$= \lim_{h\to 0} \frac{-3\cancel{h}}{\cancel{h}(\sqrt{8-3x-3h} + \sqrt{8-3x})}$$

$$= \boxed{\frac{-3}{2\sqrt{8-3x}}}$$

—or—

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sqrt{8-3x} - \sqrt{8-3a}}{x-a}$$

$$= \lim_{x\to a} \frac{\sqrt{8-3x} - \sqrt{8-3a}}{(x-a)} \cdot \frac{\sqrt{8-3x} + \sqrt{8-3a}}{\sqrt{8-3x} + \sqrt{8-3a}}$$

$$= \lim_{x\to a} \frac{(\cancel{8}-3x)-(\cancel{8}-3a)}{(x-a)(\sqrt{8-3x} + \sqrt{8-3a})}$$

$$= \lim_{x\to a} \frac{-3\cancel{(x-a)}}{\cancel{(x-a)}(\sqrt{8-3x} + \sqrt{8-3a})}$$

$$= \boxed{\frac{-3}{2\sqrt{8-3a}}}$$

4. Determine the first derivative of each of the following functions. Remember to use correct notation to write your final answer.

(a) [6 pts] $f(x) = e^2 + \dfrac{5}{x^2} + \dfrac{x}{x+1}$

$$f'(x) = \frac{-10}{x^3} + \frac{(x+1)(1)-(x)(1)}{(x+1)^2} = \frac{-10}{x^3} + \frac{1}{(x+1)^2}$$

or $f'(x) = -10x^{-3} + x(-(x+1)^{-2}) + (x+1)^{-1}(1)$

(b) [6 pts] $f(x) = (2x^5 + 3)\tan(x)$

$$f'(x) = (2x^5+3)\sec^2(x) + \tan(x)(10x^4)$$

(c) [6 pts] $f(x) = (\arcsin(3x))^2$

$$f'(x) = 2(\arcsin(3x)) \cdot \frac{1}{\sqrt{1-(3x)^2}} \cdot 3$$

(d) [8 pts] $y = x^{3x^2-x}$

$$\ln(y) = \ln(x^{3x^2-x})$$

$$\ln(y) = (3x^2-x)\ln(x)$$

$$\frac{1}{y}\frac{dy}{dx} = (3x^2-x)\cdot\frac{1}{x} + \ln(x)(6x-1)$$

$$\frac{dy}{dx} = y\left[\frac{3x^2-x}{x} + (6x-1)\ln(x)\right]$$

$$\frac{dy}{dx} = x^{3x^2-x}\left[\frac{3x^2-x}{x} + (6x-1)\ln(x)\right]$$

5. Determine the following indefinite integrals.

(a) [6 pts] $\int \left(\frac{6x^4+3x^3-8}{x}\right) dx = \int (6x^3+3x^2-\frac{8}{x})dx$

$= \frac{6}{4}x^4 + x^3 - 8\ln|x| + C = \frac{3}{2}x^4 + x^3 - 8\ln|x| + C$

(b) [6 pts] $\int \left(\cos(2x) - \frac{1}{1+x^2}\right) dx = \frac{1}{2}\sin(2x) - \arctan(x) + C$

(c) [6 pts] $\int \frac{1}{\sqrt{x}(5+2\sqrt{x})^3} dx = \int \frac{1}{\sqrt{x}} \cdot \frac{1}{(5+2\sqrt{x})^3} dx = \int \frac{1}{u^3} du = \int u^{-3} du$

$u = 5 + 2\sqrt{x}$

$du = 2 \cdot \frac{1}{2}x^{-1/2} dx = \frac{1}{\sqrt{x}} dx$

$= -\frac{1}{2}u^{-2} + C$

$= -\frac{1}{2}(5+2\sqrt{x})^{-2} + C$

6. Evaluate the following definite integrals.

(a) [6 pts] $\displaystyle\int_0^1 (x^3 + e^{-3x})\,dx$ $= \frac{1}{4}x^4 + -\frac{1}{3}e^{-3x} \Big|_{x=0}^{x=1} = \frac{1}{4} - \frac{1}{3}e^{-3} - \left(0 - \frac{1}{3}\right)$

$= \frac{3}{12} + \frac{4}{12} - \frac{1}{3}e^{-3} = \boxed{\frac{7}{12} - \frac{1}{3}e^{-3}}$

(b) [6 pts] $\displaystyle\int_0^4 \sqrt{4y+9}\,dy$ $= \int_9^{25} \sqrt{u} \cdot \frac{1}{4}\,du = \frac{1}{4} \cdot \frac{2}{3} u^{3/2} \Big|_9^{25} = \frac{1}{6} \cdot (25)^{3/2} - \frac{1}{6}(9)^{3/2}$

$u = 4y+9$
$du = 4\,dy$
$\frac{1}{4}du = dy$
$y=0 \Rightarrow u=9$
$y=4 \Rightarrow u=25$

$= \frac{5^3 - 3^3}{6} = \frac{125-27}{6} = \frac{98}{6} = \boxed{\frac{49}{3}}$

(c) [6 pts] $\displaystyle\int_0^{\pi/2} \frac{\sin x}{1+\cos x}\,dx$ $= \int_0^{\pi/2} \frac{1}{1+\cos(x)} \cdot \sin(x)\,dx = \int_2^1 \frac{1}{u} \cdot -1\,du$

$u = 1+\cos(x)$
$du = -\sin(x)\,dx$
$-du = \sin(x)\,dx$
$x=0 \Rightarrow u=2$
$x=\pi/2 \Rightarrow u=1$

$= \int_2^1 \frac{-1}{u}\,du = -\ln(u)\Big|_{u=2}^{u=1} = -\ln(1) + \ln(2)$
$= \boxed{\ln(2)}$

7. [10 pts] Water is added to an empty rain barrel at a rate of $30-2t$ gallons per hour, starting at time $t = 0$, until the tank is completely full. If the rain barrel holds 225 gallons, how long will it take to completely fill the tank?

Hint: What does $30 - 2t$ *gallons per hour* represent?

$\frac{dV}{dt} = 30-2t$ gallons per hour

$V = 225$ when full

$V = 30t - t^2 + C$

$V(0) = 0$ so $0 = 30(0) - (0)^2 + C$

$0 = C$

$V = 30t - t^2$

Barrel is full when $225 = 30t - t^2$

$t^2 - 30t + 225 = 0$

$(t-15)^2 = 0$

$t = 15$ hrs when the tank is completely full

8. [14 pts] The surface area of a cube of ice is decreasing at a rate of 10 cm^2/s. At what rate is the volume of the cube changing when the surface area is 24 cm^2?

$\frac{dV}{dt} = ?$ when $A = 24$

$\frac{dA}{dt} = -10$ cm^2/s

$V = x^3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt} = 3(2)^2\left(\frac{-5}{12}\right)$

$= \boxed{-5}$ cm^3/s

$A = 6x^2$ where x = side length

$24 = 6x^2$

$4 = x^2$

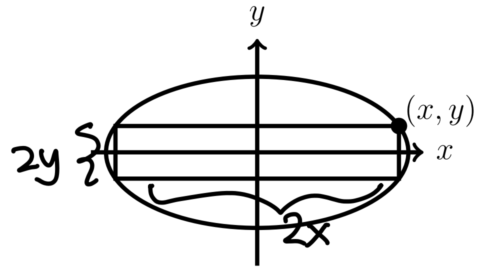
$x = 2$ at the time of interest

$\frac{dA}{dt} = 12x\frac{dx}{dt}$ so $-10 = 12(2)\frac{dx}{dt}$

$\frac{-10}{24} = \frac{-5}{12} = \frac{dx}{dt}$

units: cm^2/s

9. A rectangle is to be inscribed in the ellipse $\frac{x^2}{4} + y^2 = 1$. (See diagram below.)



$y^2 = 1 - \frac{x^2}{4}$

$y = \sqrt{1 - \frac{x^2}{4}}$

______ (a) [10 pts] Let x represent the x-coordinate of the top-right corner of the rectangle. Determine a formula $A(x)$ for the area of the rectangle as a function of x alone. Your final answer should include the variable x and can not include any other variables.

$A = bh$

$A = (2x)(2y)$

$A(x) = 4x\sqrt{1 - \frac{x^2}{4}}$

______ (b) [4 pts] Suppose you want to determine the dimensions of the rectangle that will result in the maximum possible area. Determine an appropriate domain for $A(x)$. Briefly explain your answer. (Note: We will not actually maximize $A(x)$ in this problem.)

$[0, 2]$ or $(0, 2)$

$x \geq 0$ since it can't be negative when (x,y) is the top-right corner

The largest possible x happens when $0 = 1 - \frac{x^2}{4}$

$\frac{x^2}{4} = 1$

$x^2 = 4$

$x = 2 \leftarrow$ largest possible x

10. Suppose you want to find the dimensions (in centimeters) of the open-top cylinder with volume $V = 16\pi$ cubic centimeters that has the minimum possible surface area.

    The surface area of an open-top cylinder with radius r centimeters and height h centimeters is $S = 2\pi rh + \pi r^2$ square centimeters. Since the volume is 16π cubic centimeters, we know that $16\pi = \pi r^2 h$.

    ________ (a) [4 pts] Determine the surface area S **as a function of** r. (The variable h should not be used in your answer.)

$16\pi = \pi r^2 h$ so $h = \frac{16}{r^2}$

$$S = 2\pi r\left(\frac{16}{r^2}\right) + \pi r^2 = \frac{32\pi}{r} + \pi r^2$$

    ________ (b) [4 pts] Determine an appropriate domain for the surface area function S from part (a), and explain briefly.

$(0, \infty)$

$r \geq 0$ since r is a radius; $r \neq 0$ due to $V = 16\pi$

    ________ (c) [10 pts] Use calculus techniques to determine the value of r which results in the minimum possible surface area. Write a sentence, using appropriate units, to summarize your answer.

$S = \frac{32\pi}{r} + \pi r^2$, domain: $(0, \infty)$

First derivative test

$S' = \frac{-32\pi}{r^2} + 2\pi r$

$0 = \frac{-32\pi}{r^2} + 2\pi r$

$\frac{32\pi}{r^2} = 2\pi r$

$32\pi = 2\pi r^3$

$16 = r^3$

$r = \sqrt[3]{16} = 2\sqrt[3]{2}$

$(0, \sqrt[3]{16})$	$(\sqrt[3]{16}, \infty)$
$S'(1) = -30\pi$	$S'(4) = \frac{-32\pi}{16} + 8\pi = 6\pi$
$-$	$+$
$\searrow$	$\nearrow$

S has an abs. min at $r = \sqrt[3]{16}$ cm.

- or -

Second derivative test

$S' = -\frac{32\pi}{r^2} + 2\pi r$

$0 = \frac{-32\pi}{r^2} + 2\pi r$

$\frac{32\pi}{r^2} = 2\pi r$

$32\pi = 2\pi r^3$

$16 = r^3$

$r = \sqrt[3]{16}$

$S'' = \frac{64\pi}{r^3} + 2\pi$ so $S''(\sqrt[3]{16}) > 0$

$\therefore$ S has an abs. minimum at $r = \sqrt[3]{16}$ cm.

This page is extra space for work. **Do not detach this page.** If you want us to consider the work on this page, you should print your name, instructor and class meeting time below. For the problems where you want us to look at this work, please write "see last page" next to your work on the problem page so that we know to look here.

Name (print): ______________________ Instructor (print): ______________________

Class Meeting Time: ______________