Proposed Department Syllabus for MATH 2250

Course Description (to be listed in UGA Bulletin)
We will use the derivative to understand the behavior of functions. We will discuss the limit, the derivative, and the antiderivative, both conceptually and computationally, culminating in the Fundamental Theorem of Calculus. We will use calculus concepts to model and solve problems in science and engineering, with emphasis on graphs, optimization, and basic integration.

Student Learning Outcomes for the Course
At the end of the semester, a successful student will be able to:

1. Calculate and interpret basic trends, rate, and accumulation using the limit, the derivative, and the integral, respectively.
2. Use a function's graph to:
    a. Identify increasing/decreasing behavior and critical numbers of the first or second derivative of the function
    b. Identify extrema
    c. Determine limits
    d. Identify points of continuity/discontinuity
    e. Identify asymptotes
    f. Identify points where the function is/is not differentiable
3. Use information (a formula or table and/or first or second derivative, etc.) about a function to predict:
    a. Behavior of the function and/or its first or second derivative
    b. Extrema
    c. Limits
    d. Points of continuity/discontinuity
    e. Asymptotes
    f. Points where function is/is not differentiable
    g. Area under the curve, net area under the curve, or area/net area between two curves
4. Apply calculus to solve an application problem by selecting an appropriate model, identifying an appropriate calculus technique, using the calculus technique on the model to solve the problem, and interpreting the solution in context.
5. Effectively communicate mathematics, in writing and orally, with their peers and with the course instructor.

Unit Learning Outcomes
Unit 1: Limits and the Definition of the Derivative
Upon successful completion of this unit of material, students will be able to:

1. Use a function's graph to
    a. evaluate limits graphically (one- and two-sided)
    b. determine points where a function is/is not continuous, and justify using the definition of continuity
    c. identify discontinuities by type, and justify

   d. determine any horizontal and vertical asymptotes
2. Use a formula for a function to
   a. evaluate limits (one- and two-sided) algebraically
      i. Using direct substitution ("plug-in") when appropriate
      ii. Using algebraic manipulation to eliminate indeterminate forms
      iii. Using key properties of essential functions such as absolute value, sine and/or cosine, e^x (e.g. signs, periodicity, domain, range, end behaviors)
      iv. Using appropriate reasoning about infinity and/or 0
   b. determine points where a function is/is not continuous, and justify using the definition of continuity
   c. identify discontinuities by type, and justify
   d. predict a function's vertical and horizontal asymptotes (if any) using limits, and justify your answer (checking zeroes or leading coefficients and/or powers is not enough)
3. Use the Intermediate Value Theorem to predict function behavior, including zeroes of the function.
4. Calculate and interpret secant slope (average rate of change, average velocity).
5. Use the definition of the derivative to calculate the derivative (slope of the curve, instantaneous rate of change, instantaneous velocity):
   a. at a point
   b. as a function
6. Identify points at which a function fails to be differentiable by using a graph or other information about the function
7. Determine an equation of the tangent line to a function at a point
8. Use the tangent line to approximate the behavior of the original function
9. Interpret the derivative of a function in an applied setting

Student Learning Outcomes for Unit 2: The Derivative
Upon successful completion of this unit of material, students will be able to:
1. Calculate derivatives using:
   a. appropriate combinations of basic differentiation rules: the constant rule, sum rule, difference rule, power rule, constant multiple rule, product rule, quotient rule, and chain rule.
   b. differentiation formulas for functions involving a^x, trigonometric functions, functions involving logarithms (with any base), and basic inverse trigonometric functions (arcsin, arccos, and arctan).

2. Use the definition of the derivative to prove the sum, constant multiple, and product rules for differentiation, and the differentiation formulas for sine and cosine.
3. Calculate and interpret dy/dx when y is given as an implied (i.e. *implicit*) function of x, typically provided as an equation involving both variables x and y. (SLO 3, Midterm 2) Determine special properties of the curve given by the equation:
    a. Points where the tangent line is horizontal
    b. Points where the tangent line is vertical
    c. Points where the tangent line is undefined
4. Use implicit differentiation of an appropriate inverse function to verify the differentiation formula for ln(x); use implicit differentiation of an appropriate inverse function to verify the differentiation formulas for arcsin(x), arccos(x), and arctan(x), including justification via an appropriate right triangle diagram or a Pythagorean identity
5. Apply the technique of logarithmic differentiation to a function raised to a functional power, and to complicated products, quotients, and powers of functions (and combinations of these types of functions)
6. Use the technique of implicit differentiation to solve an application problem by selecting an appropriate model, implicitly differentiating an appropriate equation representing the model to *relate rates of change*, determining any needed quantities in the resulting equation, and interpreting the solution in context
7. Determine the linearization and differential of a given function
8. Use linearization and/or the differential to solve an application problem by selecting an appropriate model, using linearization and/or differentiation on the model to solve the problem, and interpreting the solution in context.
9. Calculate and interpret velocity and acceleration in an application problem.

Student Learning Outcomes for Unit 3: Applications of the Derivative

Upon successful completion of this unit of material, students will be able to:

1. Use the mean value theorem to:
    a. Use a bound on the derivative to restrict the function's behavior
    b. Provide a rationale for the fact that if f'(x)=g'(x), then f and g differ by a constant

2. Determine intervals of increase and intervals of decrease; determine intervals where a function is concave up and intervals where it is concave down. Determine relative extrema and inflection points.
3. Solve optimization (“max/min”) problems, with function and domain provided, using the following techniques:
    a. Closed interval method for absolute extrema (Extreme Value Theorem)
    b. First derivative test for relative (local) extrema on an open interval
    c. Second derivative test for relative (local) extrema on an open interval
4. Sketch a function’s graph given a graph or information about its derivative; sketch a function’s derivative based on a graph or information about the original function
5. Sketch the graph of a function by incorporating some or all of the following information: limits, increasing/decreasing behavior and local extrema, concavity and inflection points, and asymptotes.
6. Identify a function’s relative and/or absolute extrema using the function’s graph.
7. Use L’Hopital’s rule to evaluate a limit which is an indeterminate form.

Student Learning Outcomes for Unit 4: Integration

Upon successful completion of this unit of material, students will be able to:

1. Apply calculus to solve an applied optimization (“max/min”) problem by
    a. selecting an appropriate model
    b. building an appropriate function and domain
    c. selecting and applying an appropriate optimization technique
    d. justifying that the extrema is actually absolute (and not just local), if needed, and
    e. interpreting the solution in context.
2. Calculate basic antiderivatives
3. Use finite sums to approximate areas under curves and signed area, using:
    a. An expression that can be plugged directly into a basic calculator and involves only numbers, arithmetic operations, and trigonometric or logarithmic functions, when the sum has at most 10 sub-intervals (with all terms in the sum written out explicitly), using left endpoints, right endpoints, and midpoints
    b. Sigma notation for large n and left endpoints and right endpoints

4. Interpret a finite sum area approximation to decide, and justify, whether the sum provides an over-estimate for the area, under-estimate for the area, or cannot be determined.
5. Evaluate a sum written in sigma notation by using summation rules and the summation formulas for constants, k, k^2, and k^3, summed from k=1 to k=n
6. Express the exact area under a curve as a limit of finite sums.
7. Evaluate a limit of finite sums.
8. Write a definite integral as a limit of Riemann sums.
9. Use a limit of Riemann sums to evaluate a definite integral:
    a. Over [0,b] using a limit of Riemann sums ("the definition of the definite integral") with left endpoints and right endpoints
    b. Over [a,b] using a limit of Riemann sums ("the definition of the definite integral") with right endpoints
10. Use formulas from geometry to determine definite integrals
11. Apply the Fundamental Theorem of Calculus (FTC) to:
    a. Differentiate a function provided in terms of a definite integral (FTC 1)
    b. Evaluate a definite integral when the integrand has a known antiderivative (FTC 2)
12. Use the Net Change Theorem (FTC 2) to determine and/or approximate the net change of a function over a given interval, where the function represents some sort of physical quantity (e.g. position, distance, weight, etc.).
13. Use integration by substitution to:
    a. Evaluate indefinite integrals
    b. Evaluate definite integrals
14. Use definite integrals to determine total area and net area:
    a. Between a curve y=f(x) and the x-axis
    b. Between two curves y=f(x) and y=g(x) (where both functions are given as functions of x)

Textbook

Our adopted text is *University Calculus, Early Transcendentals*, Fourth Edition. Presently most sections are using a free text available via PDF download is the UGA Custom Edition of OpenStax Calculus I, available here: https://openstax.org/details/books/calculus-volume-1 The table below includes a sample daily schedule for the recommended topics, assuming 4 class meetings per week, with 51 days of instruction, plus 4 exam review days and 4 in-class exams.

Day	OpenStax	Thomas	Topic
1	NA	NA	Course Intro (Icebreaker/Syllabus/Precal review)
2	2.1	2.1	Rates of Change and Tangents to Curves
3	2.2, 2.3	2.2	Limit of a function/limit laws
4	2.2, 2.3	2.4	One-sided limits
5		Flex	Limits - Instructor Choice (2.2/2.4)
6	2.4	2.5	Continuity
7	4.6	2.6	Limits Involving Infinity/Asymptotes
8	4.6	2.6	Limits Involving Infinity/Asymptotes
9	3.1	3.1	Tangents and the Derivative at a Point
10	3.2	3.2	The Derivative as a Function
11	3.3	3.3	Differentiation Rules
12	3.3	3.3	Differentiation Rules
13	3.4	3.4	Derivative as Rate of Change
14	3.5	3.5	Derivatives of Trig Functions
15	3.6	3.6	The Chain Rule
16		Flex	Differentiation Rules - Instructor Choice (3.3-3.6)
17	3.8	3.7	Implicit Diff
18	3.7, 3.9	3.8	Derivatives of Inverse Functions, Logs
19	3.7, 3.9	3.8	Derivatives of Inverse Functions, Logs
20	3.7	3.9	Derivatives of Inverse Trig Functions
21	4.1	3.ten	Related Rates
22	4.1	3.ten	Related Rates
23	4.1	3.ten	Related Rates
24	4.2	3.11	Linearization and Differentials
25	4.3	4.1	Extreme Values
26	4.3	4.1	Extreme Values
27	4.4	4.2	Mean Value Theorem
28	4.5	4.3	Monotonic Functions and the First Derivative Test
29	4.5	4.3	Monotonic Functions and the First Derivative Test
30	4.5	4.4	Concavity and Curve Sketching
31	4.5	4.4	Concavity and Curve Sketching
32	4.8	4.5	Indeterminate Forms and L'Hopital's Rule
33	4.8	4.5	Indeterminate Forms and L'Hopital's Rule
34		Flex	Curve Sketching - Instructor choice (4.2-4.5)
35	4.7	4.6	Applied Optimization
36	4.7	4.6	Applied Optimization
37	4.7	4.6	Applied Optimization
38	4.9	4.7	Flex - Newton's Method or Instructor choice

39	4.ten	4.8	Antiderivatives
40	4.ten	4.8	Antiderivatives
41	5.1	5.1-5.2	Areas/Finite Sum Estimates, Sigma Notation, Limits of Finite Sums
42	5.1	5.1-5.2	Areas/Finite Sum Estimates, Sigma Notation, Limits of Finite Sums
43	5.2	5.3	The Definite Integral
44	5.2	5.3	The Definite Integral
45	5.3, 5.4	5.4	The Fundamental Theorem of Calculus
46	5.3, 5.4	5.4	The Fundamental Theorem of Calculus
47	5.5	5.5	Indefinite Integrals and Substitution
48	5.5	5.6	Substitution and Areas Between Curves
49	5.5	5.6	Substitution and Areas Between Curves
50			Flex/Review (how to study for final exams)
51			Flex/Review

Classroom Style

Students learn best in an active learning environment. In a traditional lecture classroom, students watch their instructor do mathematics, but students do not learn solely from watching their instructor do math. While students must do some work outside of class, it is also essential for students to do mathematics with the assistance of the course instructor, during class time. Various resources exist for facilitating active learning in Calculus I, including flipped classroom materials; contact the Calculus I coordinator for further information.

Assessment

Instructors are encouraged to assess student understanding by carefully monitoring student work during class; while monitoring, instructors can provide helpful feedback and keep students on track for success. Instructors are encouraged to seek out correct thinking within an incorrect solution. (What part of the students' thinking could be modified to work?)

Students' written work must be assessed frequently via quizzes, in-class work, or written homework assignments. Detailed, prompt feedback must be provided following these assessments. Students should take 3-4 in-class exams followed by the cumulative mass exam.

Thinking Ahead

Near advising and registration, encourage your students to take Calculus II.