University of Georgia   **Math 1113**
Department of Mathematics   **Final Exam**   Spring 2018

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): _____   Name (print): _____

Student Number: _____

Instructor's Name: _____   Class Time: _____

- If you need extra space use the last page. *Do not tear off the last page!*

- Please show your work. **An unjustified answer may receive little or no credit.**

- If you make use of a theorem to justify a conclusion then state the theorem used by name.

- Your work must be **neat**. If we can't read it (or cannot find it), we cannot grade it.

- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.

- Please turn off your mobile phone.

- You are only allowed to use a TI-30 calculator. No other calculators are permitted.

- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

- Common identities:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta),$$
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).$$

Problem Number	Points Possible	Points Made
1	20	
2	15	
3	20	
4	15	
5	10	
6	20	
7	15	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
Total:	175	

1. Determine all values of x that satisfy each equation below. Print your answer(s) in the box provided, and **your answer(s) should be exact.** (No decimal approximations.)

_____ (a) [5 pts] $3x^2 = 5x$

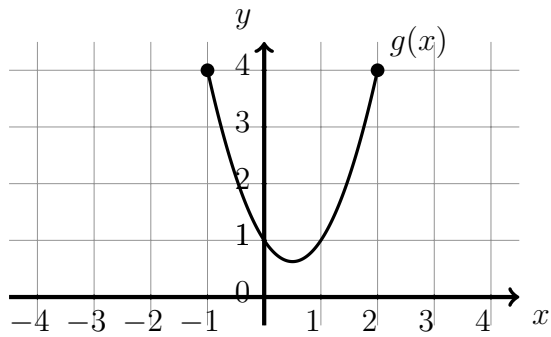$x =$

_____ (b) [5 pts] $e^{2x+1} = 8$

$x =$

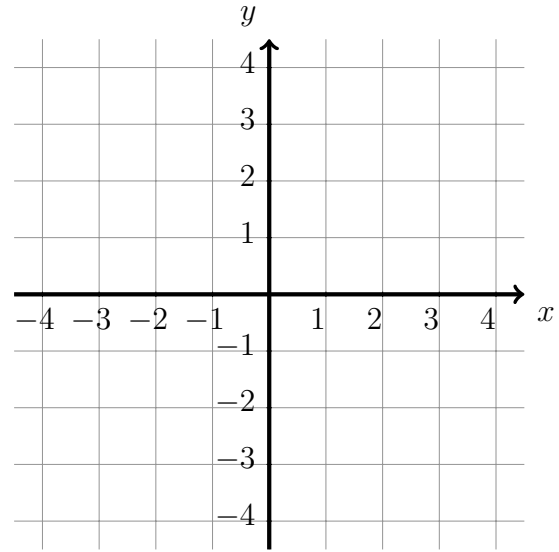(c) [5 pts] $\ln(4x + 1) = 5$
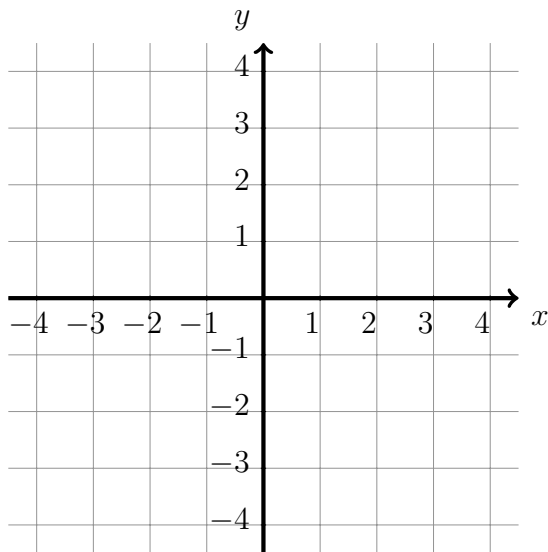
_____

$x =$

_____ (d) [5 pts] $\tan(3x + 1) = \sqrt{3}$
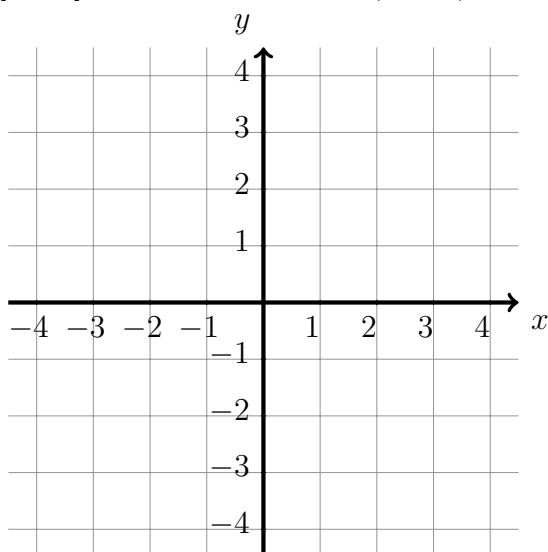(Determine only the smallest, positive value of x, and your answer should be in radians.)

$x =$

2. Given the plot of $g(x)$ in the following figure, make a sketch of each of the functions requested below.

(a) [5 pts] Make a sketch of $\frac{1}{4}g(x)$.

(c) [5 pts] Make a sketch of $-g(2x)$.

(b) [5 pts] Make a sketch of $g(x + 1) - 3$.

Points earned: _____
out of a possible 15 points

3. Answer each of the questions below. The functions f and g are defined using the following tables. If a value does not exist briefly state why the value does not exist.

t	$f(t)$
0	2
1	-2
2	3
3	5

t	$g(t)$
0	6
1	3
2	-3
3	5

(a) [5 pts] Determine the value of $f(g(1))$.

(b) [5 pts] Determine the value of $g(f(1))$.

(c) [5 pts] Determine the value of $g^{-1}(f(2))$.

(d) [5 pts] Determine the domain of $g(f(t))$.

4. For each scenario below circle the phrase that best describes the **kind** of function that will best approximate the phenomena described.

(a) [5 pts] The amount of money in a bank account as a function of time. The interest is compounded monthly, and there are no withdrawals.

| Linear Function | Quadratic Function | Exponential Function | Logistic Function | Trigonometric Function |

(b) [5 pts] The cost of an option increases in time at a constant rate.

| Linear Function | Quadratic Function | Exponential Function | Logistic Function | Trigonometric Function |

(c) [5 pts] The height of a pendulum that swings back and forth.

| Linear Function | Quadratic Function | Exponential Function | Logistic Function | Trigonometric Function |

5. [10 pts] Verify that the relationship

$$\frac{\sin(\omega) - \csc(\omega)}{\csc(\omega)} = -\cos^2(\omega)$$

is an identity. Show your work and justify your conclusions.

6. For each situation below determine an equation that satisfies the criteria. In each case use proper notation. (If the resulting function is a piecewise defined function, then you should use the correct notation for a piecewise defined function.)

_____ (a) [5 pts] Circle of radius 4 centered at $(8, -3)$.

_____ (b) [5 pts] A function defined for $0 \le x \le 6$. For $0 \le x \le 2$ the function is a line that has slope 2 and goes through the point $(1, 4)$. For the rest of the domain the function is a line that goes through the points $(3, 1)$ and $(5, 4)$.

_____ (c) [10 pts] A cosine function, $y = A\cos(bx + c) + d$, is shown below, where A, b, c, and d are constants. If A and b are positive numbers determine a formula for the function. (Each square is one unit wide and one unit high.)
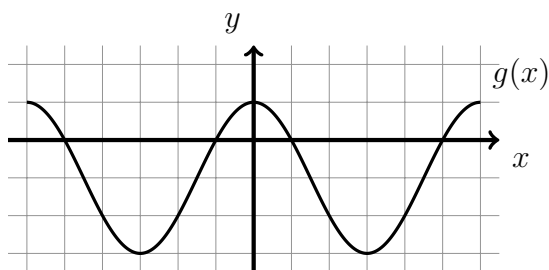
7. The following questions refer to the function

$$h(x) = e^{3x} - 8.$$

_____ (a) [5 pts] Determine the inverse of the function.

(b) [5 pts] Determine the range and the domain of the inverse of h.

_____

(c) [5 pts] Express the function as a composition of two other functions so that $h(x) = f(g(x))$. Your choice should not be $f(x) = x$ nor $g(x) = x$.

_____

8. [10 pts] A vial of radioactive material is found at a site where it was left unattended. At the time the vial was found there was 10 grams of material. Four days later it is estimated that there is 8 grams of material. Determine how much material will be in the vial twelve days after it was found.

9. Answer each of the following.

_____ (a) [5 pts] Determine the exact numerical value of $\arccos\left(\cos\left(\dfrac{5\pi}{3}\right)\right)$.

(b) [5 pts] Determine an alternate formula for $\tan\left(\arcsin\left(x\right)\right)$ that does not contain any trigonometric functions.

10. [10 pts] A helicopter will take off from a landing pad, and it will rise straight up at a constant speed of four meters per second. A camera will be placed 200 meters away from the pad and will be aimed at the helicopter. What should the angle of elevation of the camera be 30 seconds after the helicopter takes off?

11. A Ferris wheel has a radius of 20 meters, and the center of the wheel is 23 meters off the ground. At the initial time a seat is at the bottom of the wheel.

    (a) [5 pts] A couple sit in the bottom seat, and the wheel is rotated $\frac{2\pi}{3}$ radians counter-clockwise. What is the seat's height above the ground?

    (b) [5 pts] The Ferris wheel moves with a constant rate of revolution, and it makes one revolution every 3 minutes. Determine the height above the ground of the seat at any time.

12. [10 pts] A car travels along a circle and makes an angle of 23° (degrees) around the circle. The inside wheel travels a distance of 4m, and the outside wheel travels a distance of 4.8m. How wide is the car?

13. [10 pts] Determine the numbers whose sum is 30 and their product is as large as possible. (Show your work and justify your answers mathematically.)

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____