Topology Prelim, Winter 2001

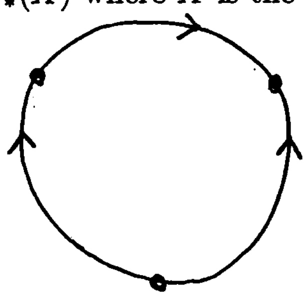
1. Let C be the union of line segments in $\mathbb{R}^2$ from $(0,1)$ to $(1/n,0)$ for $n=1,2,3,\ldots$ and the line segment from $(0,1)$ to $(0,0)$. Show that there is no contraction of C that keeps $(0,0)$ fixed at all times.

2. How many path components does the group of 2×2 invertible matrices, $Gl(2,\mathbb{R})$ have?

3. Let X and Y be metric spaces and let $f: X\times I \to Y$ be a homotopy. Define $D(t)$ to be the diameter of the set $f_t(X)$ (that is, it is the supremum of the distances between pairs of elements in the subset $f_t(X)$ of Y). Show that if X is compact then D is continuous. Show by example that without this assumption, D may not be continuous even if Y is compact.

4. Let A be a submanifold of a manifold M. Give necessary and sufficient conditions for the existence of a covering space map $p: M'\to M$ such that $p^{-1}(A)$ is not connected.

5. Compute $H_*(X)$ where X is the disk D^2 with three arcs identified as drawn below.



6. Let S be a surface of genus 2. Show that the natural map $\pi_1(S,*)\to H_1(S)$ has a non-trivial kernel.

7. Let $S^1=\{v\in\mathbb{R}^2 \mid |v|=1\}$. Suppose $f: S^1\to\mathbb{R}^2-0$ is continuous and $v\cdot f(v)>0$ for all $v\in S^1$. Prove that f does not extend to a function from D^2 into $\mathbb{R}^2-0$.

8. Prove that S^1 is not contractible. (Give a fairly detailed sketch of any facts you use.)