Topology Preliminary Examination
August, 2009

Instructions: Work all problems. Give clear explanations and complete proofs.

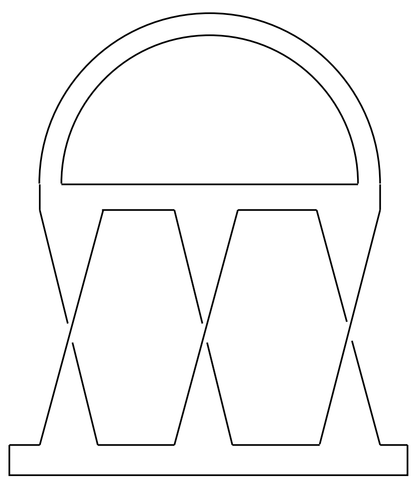
(1) (a) Show that a path-connected space is connected.
(b) A topological space X is *locally path-connected* if for every point $x \in X$ and every neighbourhood V of x, there is a path-connected open set U with $x \in U \subset V$. Show that a connected and locally path-connected space is path-connected.

(2) A topological space X is *normal* if it is Hausdorff, and, for any pair of disjoint closed sets $A, B \subset X$, there are disjoint open sets $U, V \subset X$ with $A \subset U$ and $B \subset V$. Let A be a closed subspace of a normal space X. Show that A and the quotient X/A are normal.

(3) Let $X = (0,1) \times (0,1)$, $Y = [0,1) \times [0,1)$, and $Z = [0,1] \times [0,1]$. Show which, if any, of X, Y, and Z are homeomorphic.

(4) Let X be the topological space obtained by identifying 3 distinct points on S^2. Calculate $H_*(X, \mathbb{Z})$.

(5) State the classification theorem for compact surfaces and identify this surface on your list:



(6) Find all homotopy classes of maps from $S^1 \times D^2$ to itself such that every element of the homotopy class has a fixed point.

(7) Find the universal cover of $RP^2 \times S^1$ and explicitly describe its group of deck transformations.

(8) Let $f : S^1 \to S^1$ by $f(z) = z^2$. Let $X = (S^1 \times [0,1])/(z,1) \sim (f(z),0)$. Give an explicit CW decomposition, with attaching maps, for X, and use it to compute $\pi_1(X)$ and $H_1(X)$.