MATH 3100 - Sequences and Series
Departmental Syllabus

As indicated by the course number, this is one of our transitional courses. The course goal is to gently move students from the computational orientation they encountered in calculus to the more rigorous approach they will need in 4000 level courses. The key word here is *gently*. Students at this level are not ready for a pure definition-theorem-proof approach. Rather, they should gradually be convinced of the value of precise definitions and careful notation, and constantly asked to construct their own concrete examples and counterexamples. By the end of the course, students should be able to write epsilon-N limit proofs, organize the estimates necessary in applying Taylor's Theorem, and have some feeling for the meaning of uniform convergence.

The prerequisite for the course is MATH 2260, Calculus for students in science and engineering. The current 2260 syllabus does include treatment of some sequence and series topics, e.g., application of various series convergence tests. You should not skip such topics in 3100, but rather take advantage of students' prior exposure to help them understand why the techniques they already know actually work.

Students can take 3000 (linear algebra), 3100, and 3200 (introduction to higher mathematics) in any order. This means you cannot assume prior exposure to induction or other proof writing techniques at the beginning of the course. (In the past, students taking 3000 and/or 3200 before 3100 have done better overall, but that may have changed now that 2260 covers some material on sequences and series.)

Just before students start meeting with their academic advisers to register for the following term, you should spend some time in class mentioning possible sequels to 3100.Course notes for MATH 3100, available online and suitable for use as texts:

- by Malcolm Adams, at http://www.math.uga.edu/~adams/SANDS807.pdf
- by Ed Azoff, at http://www.math.uga.edu/~azoff/courses/3100s05.pdf; latest course homepage at http://www.math.uga.edu/~azoff/courses/3100.html.

Topic Summary

Topic	**Comments**	**Adams Sections**
Ordered fields	Axiomatic treatment is optional, but developing computational facility with inequalities and absolute values is a must	1.1prob12, 1.4
Completeness and the reals	This can be based on least upper bounds (Azoff) or on monotone sequences (Adams)	1.6
Definition of sequence, examples, recursion and induction	Newton's method provides a good example of recursion.	1.1,1.2,1,3
Definition of sequence convergence (epsilon-N)	Do not rush this.	1.4
Algebra of sequential limits	Some of these results can be stated without proof.	1.5

Monotone sequences	Regardless of whether convergence of bounded monotone sequences is taken as an axiom (Adams) or based on least upper bounds (Azoff), it is important for students to have an intuitive feeling for this.	1.6
Subsequences	Use honest function notation to emphasize the fact that a subsequence is a composite of functions.	1.3
Cauchy sequences	This concept (and the preceding one) can be applied in a negative way to establish divergence of sequences.	1.6
Applications of l'Hopital's rule	Use of calculus results makes this course less self-contained, but the computational benefits are well-worth the sacrifice.	1.5
Applications to calculus	This topic can be postponed till later in the course if desired. The idea is to expose students to J. Hollingsworth's "three C's": continuity, connectedness (the intermediate value theorem), and compactness (the maximum value theorem)	1.7
Definition of series convergence	The distinction between sequences of terms and sequences of partial sums needs to be emphasized repeatedly.	2.1
Geometric and telescoping series	Use these to illustrate the preceding definition.	2.1
Series convergence tests (n'th term, comparison, limit-comparison, integral, root, ratio)	Students should have some prior experience with the mechanics of these tests, so the emphasis should be on why they work.	2.2
Absolute and conditional convergence	Include an informal discussion of series rearrangement.	2.3
Mixed convergence problems	Hopefully, this can be covered relatively quickly because of MATH 2260	2.3
Taylor's theorem (theory)	While you will probably want to include a proof, it is most important for students to understand what the Theorem says.	3.2
Taylor's theorem (computations and estimates)	Move from simple (find specified Taylor polynomials) to intermediate (estimate errors of specified approximations) to "advanced" (decide how many terms are necessary for given accuracy).	3.2
Power series: domains of convergence	Do not emphasize formulas for radii of convergence, but have students apply convergence tests for numerical series.	2.4
Uniform convergence and continuity	This should be regarded as a capstone for the course.	3.1
Operations on power series and applications to Taylor series	You may go easy on these proofs, but be sure students can get Taylor series for functions like arctan(x^3) from the series for 1/(1+x).	3.2
Optional	Decimal expansions, complex series and Euler's formula.	3.3