

**Kossack**  
**Spring 2015**

1. Suppose that  $\lim_{x \rightarrow a} f(x) = b$  and  $\lim_{x \rightarrow a} g(x) = c$ . Find  $\lim_{x \rightarrow a} f(x)g(x)$ .
2. A tangent line is drawn to the hyperbola  $xy = c$  at the point  $(a, c/a)$ . Show that the area bounded by the coordinate axes and the tangent line is independent of  $a$  and find this area.
3. Show that at each point where the curves  $y = ax^3$  and  $x^2 + 3y^2 = b$  intersect, their tangent lines are perpendicular.
4. For what values of  $c$  does the equation  $\ln x = cx^2$  have exactly one solution?
5. Sketch the graph of  $y = x - \tan^{-1}(x)$ . Show that the graph has 2 slant asymptotes and find them.
6. Find the area of the largest rectangle that can be inscribed in the ellipse  $x^2/a^2 + y^2/b^2 = 1$ .
7. A cone is inscribed in a larger cone of height 1, so that its vertex is at the center of the larger cone. Show that the inner cone has maximum volume when its height is  $1/3$ .
8. If  $x \sin(\pi x) = \int_0^{x^2} f(t) dt$ , where  $f(x)$  is continuous, find  $f(4)$ .