Study Guide for Algebra Qualifying Exam

Group Theory
> subgroups and quotient groups
> Lagrange's Theorem
> fundamental homomorphism theorems
> group actions with applications to the structure of groups such as
> > the Sylow Theorems
> group constructions such as:
> > direct and semi-direct products
> structures of special types of groups such as:
> > p-groups
> > dihedral, symmetric and alternating groups, cycle decompositions
> > the simplicity of A_n, for $n \geq 5$
> free groups, generators and relations
> solvable groups

References: [1,3,4]

Linear Algebra
> determinants
> eigenvalues and eigenvectors
> Cayley-Hamilton Theorem
> canonical forms for matrices
> linear groups (GL_n, SL_n, O_n, U_n)
> dual spaces, dual bases, induced dual map, double duals
> finite-dimensional spectral theorem

References: [1,2,4]

Foundations
> Zorn's Lemma and its uses in various existence theorems such as that of a basis
> for a vector space or existence of maximal ideals.

References: [1,3,4]

Theory of Rings and Modules
> basic properties of ideals and quotient rings
> fundamental homomorphism theorems for rings and modules
> characterizations and properties of special domains such as:
> > Euclidean implies PID implies UFD
> classification of finitely generated modules over PIDs with emphasis on Euclidean
> domains
> applications to the structure of:
> > finitely generated abelian groups
> > canonical forms of matrices

References: [1,3,4]

Field Theory
>   algebraic extensions of fields
>   fundamental theorem of Galois theory
>   properties of finite fields
>   separable extensions
>   computations of Galois groups of polynomials of small degree and cyclotomic
>   polynomials
>   solvability of polynomials by radicals

References: [1,3,4]

As a general rule, students are responsible for knowing both the theory (proofs) and practical applications (e.g. how to find the Jordan or rational canonical form of a given matrix, or the Galois group of a given polynomial) of the topics mentioned. A supplement to this study guide is available at:
http://www.math.uga.edu/sites/default/files/PDFs/Graduate/QualsStudyGuides/AlgebraPhDqualr emarks.pdf

References

[1] David Dummit and Richard Foote, *Abstract Algebra*, Wiley, 2003.
[2] Kenneth Hoffman and Ray Kunze, *Linear Algebra*, Prentice-Hall, 1971.
[3] Thomas W. Hungerford, *Algebra*, Springer, 1974.
[4] Roy Smith, *Algebra Course Notes* (843-1 through 845-3), http://www.math.uga.edu/~roy/, 1996.

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