University of Georgia **Math 1113**
Department of Mathematics **Final Exam** Fall 2018

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): _____ Name (print): _____

Student Number: _____

Instructor's Name: _____ Class Time: _____

Problem Number	Points Possible	Points Made
1	20	
2	10	
3	30	
4	15	
5	10	
6	15	
7	10	
8	5	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
Total:	175	

- If you need extra space use the last page. *Do not tear off the last page!*

- Please show your work. **An unjustified answer may receive little or no credit.**

- If you make use of a theorem to justify a conclusion then state the theorem used by name.

- Your work must be **neat**. If we can't read it (or cannot find it), we cannot grade it.

- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.

- Please turn off your mobile phone.

- You are only allowed to use a TI-30 calculator. No other calculators are permitted.

- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

- Common identities:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta),$$
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).$$

1. Determine all values of x that satisfy each equation below. Print your answer(s) in the box provided.

    (a) [5 pts] $x^2 = 5 - 3x$

$x =$

    (b) [5 pts] $7 \cdot 3^{6x+1} = 4$

$x =$

out of a possible 10 points

(c) [5 pts] $\sin(x) = 0.6$ where x is in the second quadrant and $0 \le x \le 2\pi$. (Your answer should be in radians and should approximate the answer to within two decimal places.)
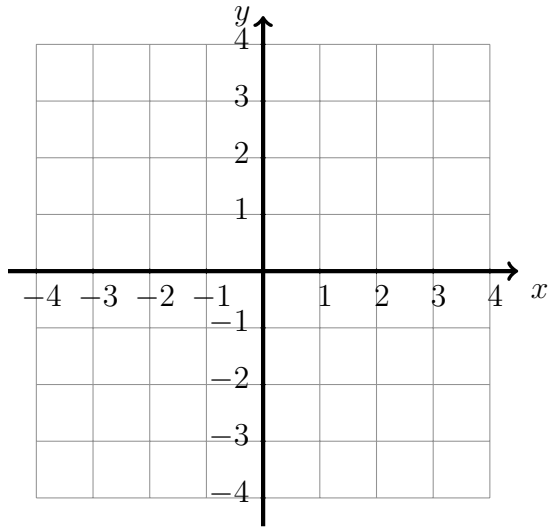
$x =$

(d) [5 pts] $\ln(4x + 1) = 2 + \ln(1 - x)$

$x =$

2. Answer each of the questions below and use the axes provided for any plots.

   (a) [5 pts] Determine a formula for the line that goes through the points $(-4, -2)$ and $(3, 1)$. Plot the two points and plot the line on the coordinate axes below.

Formula:

   (b) [5 pts] Determine a formula for the line through the point $(-1, 3)$ that has a slope of -2. Plot the line on the coordinate axes below.

Formula:

out of a possible 10 points
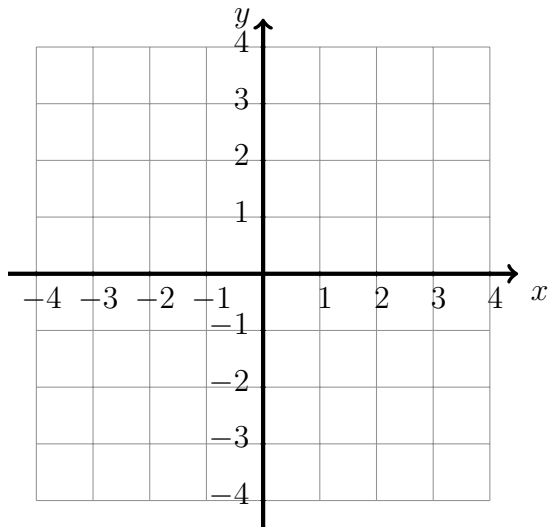
3. The graph of a function, h, is given in the plot below, and a table is given for all of the values of a function, g. Use the information in the graph and the table to answer each of the questions below. **If a value does not exist briefly explain why.**

Graph of $h(x)$
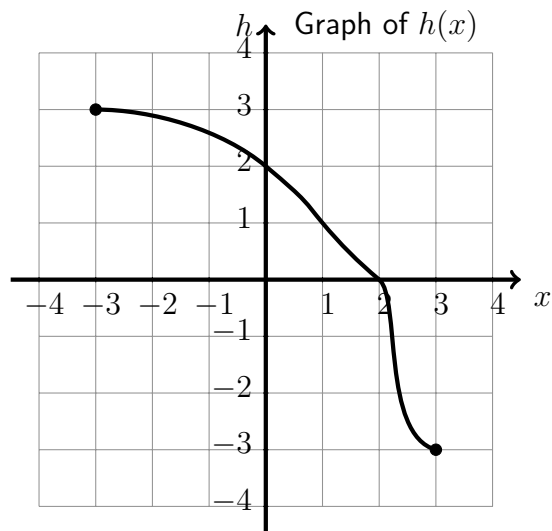
x	$g(x)$
0	5
1	0
2	-3
3	1
4	3

(a) [5 pts] Is the function h 1-1? (Briefly justify your answer.)

(b) [5 pts] Determine the value of $g(h(2))$.

(c) [5 pts] Determine the value of $h(g(2))$.

(d) [5 pts] Determine the value of $g^{-1}(h(3))$.

(e) [5 pts] Determine the value of $g(h^{-1}(3))$.

(f) [5 pts] Determine the domain of h.

4. For each scenario below circle the phrase that best describes the **kind** of function that will best approximate the phenomena described.

_____ (a) [5 pts] A car's velocity where the velocity increases at a constant rate as a function of time.

| Linear | Quadratic | Exponential | Logistic | Trigonometric |
| Function | Function | Function | Function | Function |

_____ (b) [5 pts] The height of a seat as a function of time that is on a ferris wheel turning at a constant rate.

| Linear | Quadratic | Exponential | Logistic | Trigonometric |
| Function | Function | Function | Function | Function |

_____ (c) [5 pts] Amount of radioactive material in a sample as a function of time.

| Linear | Quadratic | Exponential | Logistic | Trigonometric |
| Function | Function | Function | Function | Function |

5. Find equivalent formulas for each of the expressions below. Your formula should not include any trigonometric functions and should be exact (no decimal answers).

_____ (a) [5 pts] $\tan\left(\arccos\left(\dfrac{7}{10}\right)\right)$

_____ (b) [5 pts] $\sin\left(\pi + \arctan(x)\right)$

6. Answer each of the following questions about the average rate of change of a function.

   (a) [6 pts] Determine the average rate of change of $k(x) = 8x^2 + 5$ from $x = -1$ to $x = 1$.

   (b) [6 pts] Determine the formula for the average rate of change of $l(x) = 3x + 1$ from $x = a$ to $x = a + 0.1$. Expand the expression and simplify as much as possible.

   (c) [3 pts] Given a function $w(x) = e^x$ show that the average rate of change of the function from $x = a$ to $x = a + 0.1$ is $e^a \cdot \frac{e^{0.1} - 1}{0.1}$.

Points earned: _____
out of a possible 15 points

7. Express each of the following polynomials in factored form.

   (a) [5 pts] $x^2 - 2x - 8$

   (b) [5 pts] $x^3 - 6x^2 - 31x + 120$ (Note: one of the roots of the polynomial is $x = 8$.)

8. [5 pts] Determine if the function

$$f(x) = 2\cos(x) + 1$$

is 1-1. Justify your conclusions.

Points earned: _____
out of a possible 5 points

9. [10 pts] As a Halloween prank, a group of irresponsible scientists hand out samples of Francium 223 (FR-223) to small children, and each sample has 4g of FR-223. The half-life of FR-223 is 1320 seconds, and it is estimated that the children will get back home 1 hour later (3600 seconds). How much FR-223 will be in each sample when the kids get home and open their prize?

10. [10 pts] One-thousand dollars will be deposited into a bank account. The interest is compounded monthly, and there will not be any withdrawals. What should the interest rate be so that the amount of money in the account will double in eighteen years?

11. The radius of the circle in the figure below is 0.33 meters, and the area of the sector (the light gray area) is 0.31 meters². (The figure is not drawn to scale.)
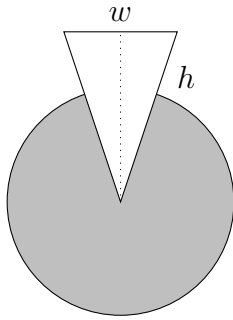
—————— (a) [5 pts] Determine the angle of the sector in radians.

(b) [5 pts] The value of w is 0.2 meters. Determine the value of h. (h is the full length of one side of the triangle.)

——————

Points earned: ——————
out of a possible 10 points

12. The virulence of a contagion is a measure of how aggressive the contagion is, and it is a number denoted by v. The time a host takes to respond to the contagion is a linear function of the virulence and is measured in hours. A group of ecologists is asked to examine a particular contagion that is infecting a local deer population.

_____ (a) [5 pts] The response time for a deer exposed to the contagion is 42 hours when $v = 0.5$ The response time decreases by 22 hours when v increases by 0.2. Determine the response time as a linear function of the virulence, v.

_____ (b) [5 pts] The fitness of the contagion is defined to be v multiplied by the response time. Determine the value of the virulence that will maximize the fitness. Briefly justify why your value of v results in the maximum and not the minimum.

Points earned: _____
out of a possible 10 points

13. [10 pts] The water level at Jeffries Creek varies between 2.4 meters and 0.2 meters due to the changes in the tide. The time between high tides is 700 minutes. The next high tide will occur at 150 minutes after midnight. Express the water level as a sine function assuming that $t = 0$ minutes is midnight.

14. [10 pts] Farmer Bob will spend \$12,000 on seed to plant in one of his fields. He figures watercress seed costs \$120 per kilogram and rutabaga seed costs \$60 per kilogram. Farmer Bob estimates that if he plants x kilograms of watercress seed he will earn x^2 in profit. He also estimated that if he plants y kilograms of rutabaga seed he will earn $200y - y^2$ in profit. How many kilograms of watercress and rutabaga seeds should Farmer Bob purchase to maximize his profit?

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____